

S.No. : 575

BAS 401

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Following Paper ID and Roll No. to be filled in your Answer Book.

PAPER ID : 09915

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B. Tech. Examination 2021-22

(Even Semester)

MATHEMATICS - III

Time : Three Hours]

[Maximum Marks : 60

Note :- Attempt all questions.

1. Attempt any four parts of the following : $5 \times 4 = 20$

(a) Show that the complex variable function $f(z) = |z|^2$ is differentiable only at origin.

(b) Define a harmonic function. Show that the function :

$$u(x, y) = x^4 - 6x^2y^2 + y^4$$

is harmonic.

(c) If $|f(z)|$ is constant, prove that $f(z)$ is also constant.

[P. T. O.]

- (d) Find the point where the Cauchy-Riemann equations are satisfied for the function

$$f(z) = x y^2 + i x^2 y$$

- (e) Find the value of the integral

$$\int_C (x + y) dx + x^2 y dy \text{ along } y = x^2$$

having (0, 0), (3, 9) end points.

- (f) Use Cauchy's integral formula to calculate :

$$\int_C \frac{2z+1}{z^2+z} dz$$

where C is $|z| = \frac{1}{2}$.

2. Attempt any four parts of the following : $5 \times 4 = 20$

- (a) Find the Taylor series expansion of a function of the complex variable :

$$f(z) = \frac{1}{(z-1)(z-3)} \text{ about the point } z = 4$$

- (b) Expand :

$$f(z) = \frac{z}{(z-1)(2-z)}$$

in Laurent series valid for $|z-1| > 1$.

- (c) Discuss singularity of:

$$\frac{1}{1 - e^z} \text{ at } z = 2\pi i$$

- (d) Determine the pole and residue at the pole of the function:

$$f(z) = \frac{z}{z-1}$$

- (e) Using residue theorem, evaluate the integral:

$$\int_C \frac{1+z}{z(2-z)} dz$$

where C is the circle $|z| = 1$.

- (f) Using complex variable techniques evaluate the integral:

$$\int_0^{2\pi} \frac{1}{5 - 3 \cos \theta} d\theta$$

3. Attempt any two parts of the following: $10 \times 2 = 20$

- (a) Calculate the first four moments of the following distribution about the mean and hence find β_1 and β_2 :

x	0	1	2	3	4	5	6	7	8
f	1	8	28	56	70	56	28	8	1

[P. T. O.]

(b) Two lines of regression are given by :

$$5y - 8x + 17 = 0 \text{ and } 2y - 5x + 14 = 0$$

If $\sigma_y^2 = 16$, find :

(i) The mean values of x and y

(ii) σ_x^2

(iii) The coefficient of correlation between x and y

(c) (i) Fit the straight line to the following data :

x	0	1	2	3	4
y	1	1.8	3.3	4.5	6.3

(ii) An urn contains 10 black and 10 white balls. Find the probability of drawing two balls of the same colour.

4. Attempt any two parts of the following : $10 \times 2 = 20$

(a) The data given below are the number of defectives in 10 samples of 100 items each construct an np-chart and comment on the results :

Sample No.	No. of defectives
1	6
2	16
3	7
4	3
5	8
6	12
7	7
8	11
9	11
10	4

(b) In a sample of 1000 cases, the mean of a certain test is 14 and standard deviation is 2.5. Assuming the distribution to be normal, find :

- (i) How many students score between 12 and 15?
- (ii) How many score above 18?
- (iii) How many score below 8?

[P. T. O.]

(c) The number of accidents in a year involving taxi drivers in a city follows Poisson distribution with mean to 3 out of 1000 taxi drivers, find approximately the number of drives with :

(i) No accidents

(ii) More than 3 accidents in a year

5. Attempt any two parts of the following : $10 \times 2 = 20$

(a) Find the Fourier transform of :

$$f(x) = \begin{cases} 1 - x^2, & \text{if } |x| \leq 1 \\ 0, & \text{if } |x| > 1 \end{cases}$$

and use it to evaluate :

$$\int_0^{\infty} \left(\frac{x \cos x - \sin x}{x^3} \right) \cos \frac{x}{2} dx$$

(b) Find Fourier cosine transform of :

$$\frac{1}{1 + x^2}$$

(c) Solve the difference equation :

$$6y_{k+2} - y_{k-1} - y_k = 0, y(0) = 0, y(1) = 1$$

by Z-transform.
