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Following Paper ID and Roll No. to be filled in your Answer Book.

PAPER ID : 29906 Roll No.

B. Tech. Examination 2021-22

(Even Semester)

DIFFERENTIAL EQUATIONS AND FOURIER ANALYSIS

Time : Three Hours] *[Maximum Marks : 60*

Note :– Attempt all questions.

SECTION–A

1. Attempt all parts of the following : $8\times1=8$

(a) Find the order and degree of the differential equation :

$$\left(\frac{dy}{dx}\right)^{1/2} + y = x^{1/3}$$

(b) Find the particular integral of the differential equation :

$$\frac{dy^2}{dx^2} + a^2 y = \sin ax$$

(c) Evaluate :

$$\int_{-1}^{1} [P_n(x)]^2 \, dx$$

(d) Express $J_3(x)$ in terms of $J_1(x)$ and $J_2(x)$.

(e) If $f(x) = 1$ is expanded in a Fourier sine series in $(0, \pi)$, then find the value of b_n.

(f) Write Dirichlets conditions for a Fourier series.

(g) Form the partial differential equation from $z = (x + a)(y + b)$, a and b being constants.

(h) Classify the equation :

$$u_{xx} + 3u_{xy} + u_{yy} = 0$$

SECTION – B

2. Attempt any two parts of the following : $2 \times 6 = 12$

(a) Apply method of variation of parameters to find the general solution of :

$$\frac{dy^2}{dx^2} - y = \frac{2}{1 + e^x}$$

(b) Find the power series solution of :

$$(1 - x^2) y'' - 2x y' + 2y = 0$$

about $x = 0$.

(c) Find the Fourier series to represent the function f (x) gien by :

$$f(x)=\begin{cases}-k, & -\pi<x<0\\ K, & 0<x<\pi\end{cases}$$

Hence show that :

$$1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\ldots\ldots\ldots=\frac{\pi}{4}$$

(d) Find the temperature in a bar of length 2 whose ends kept at zero and lateral surface insulated if the initial temperature is :

$$\sin\frac{\pi x}{2}+3\sin\frac{5\pi x}{2}$$

SECTION – C

Note :– *Attempt all questions. Attempt any two parts from each questions.* 5×8=40

3. *(a) Solve the differential equation :*

$$\frac{dy^2}{dx^2}+y=\sin x\sin 2x$$

(b) Solve :

$$x^2\frac{dy^2}{dx^2}-2\left(x^2+x\right)\frac{dy}{dx}+\left(x^2+2x+2\right)y=0$$

by removing the first derivative.

(c) Solve :

$$Dx + Dy + 3x = \sin t$$
$$Dx + y - x = \cos t$$

4. (a) Prove that :

$$J_{-n}(x) = (-1)^n J_n(x)$$

where n is a positive integer.

(b) Prove that :

$$P_n(x) = \frac{1}{2^n n!} \frac{d^n}{dx^n} \left(x^2 - 1\right)^n$$

(c) Show that :

$$x J_n' = n J_n - x J_{n+1}$$

5. (a) Find the Fourier series representing :

$$f(x) = x, \quad 0 < x < 2\pi$$

(b) Find the Fourier half-range cosine series of the function :

$$f(x) = \begin{cases} 2x, & 0 < x < 1 \\ 2(2-x), & 1 < x < 2 \end{cases}$$

(c) Find the Fourier series expansion for the function :

$$f(x) = x \cos x, \quad -\pi < x < \pi$$

6. (a) Solve :

$$4 \frac{\partial^2}{\partial}$$

(b) Solve :

(D

(c) Using

solve

6. (a) Solve :

$$4\frac{\partial^2 z}{\partial x^2} - 4\frac{\partial^2 z}{\partial x\,\partial y} + \frac{\partial^2 z}{\partial y^2} = e^{x+2y}$$

(b) Solve :

$$(D + 1)(D + D' - 1)\,Z = \sin(x + 2y)$$

(c) Using the method of separation of variables, solve :

$$\frac{\partial^2 z}{\partial x^2} - 2\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = 0$$

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